Seat No. _____

# HB-003-1016003

## B. Sc. (Sem. VI) Examination

**April - 2023**

**Mathematics : Paper - 10 (A)**

***(Optimization & Numerical Analysis - II)***

**Faculty Code : 003**

**Subject Code : 1016003**

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

**Instruction:** Answer all five questions.

**1** (a) Answer the following questions : **4**

(1) Explain in your words : Objective function with respect to Linear Programming Problems.

(2) Explain in your words : Hyperplane in $E^n$.

(3) Explain in your words : Convex set.

(4) Explain in your words : Convex Hull.

(b) Answer any **one** : **2**

(1) Write matrix form of Linear Programming Problems.

(2) Explain in your words : Slack variables and Surplus variables.

(c) Answer any **one** : **3**

(1) Draw a sketch of the graph of following LPP :

Maximize $Z = 5x_1 + 7x_2$

Subject to

$x_1 + x_2 \le 4,\ 3x_1 + 8x_2 \le 24,\ 10x_1 + 7x_2 \le 35,\ x_1, x_2 \ge 0$

(2) Explain steps of Graphical method to solve Linear Programming Problems.

(d) Answer any **one** : **5**

(1) Explain Simplex method to solve Linear Programming Problems.

(2) Explain Big-M method to solve Linear Programming Problems.

**2** (a) Answer the following questions : **4**

(1) Explain in your words : NWCM in one line.

(2) Explain in your words : LCM in one line.

(3) Explain in your words : VAM in one line.

(4) Give the name of the method to solve assignment problems.

(b) Answer any **one** : **2**

(1) What is mathematical form of transportation problem?

(2) Explain Matrix Minima Method.

(c) Answer any **one** : **3**

(1) Write three steps of NWCM.

(2) Write six relationships between primal and dual LP problems.

(d) Answer any **one** : **5**

(1) Explain penalty method with an example.

(2) Obtain the dual problem of the following primal LP problem :

Minimize $Z_x = 5x_1 + 2x_2 + x_3$

subject to

$x_1 - 3x_2 + 4x_3 = 5,\ x_1 - 2x_2 + 0x_3 \leq 3,\ 0x_1 + 2x_2 - x_3 \geq 4$

and $x_1, x_2 \geq 0, x_3$ is unrestricted.

**3** (a) Answer the following questions : **4**

(1) Write Gauss forward interpolation formula.

(2) Write Gauss backward interpolation formula.

(3) Write Sterling formula.

(4) Write Bessel's formula.

(b) Answer any **one** : **2**

(1) Write the formula which is used for inverse interpolation.

(2) Write a relation of forward difference operator in terms of central difference operator and shifting operator and then write central difference operator in terms of shifting operator and forward difference operator.

(c) Answer any **one** : **3**

(1) Obtain Laplace-Everett's formula.

(2) If $f(x) = x^3$ then find $f(1, 3, 5, 7)$.

(d) Answer any **one** : **5**

(1) Use Lagrange's formula to find the form of $f(x)$ given

| $x$ | 0 | 2 | 3 | 6 |
|---|---|---|---|---|
| $f(x)$ | 648 | 704 | 729 | 792 |

(2) Obtain the formula to interpolate the value of $y$ for $0 < P < 1$.

**4** (a) Answer the following questions : **4**

(1) What is numerical differentiation?

(2) Fill in the blank :

To find $\int_a^b y dx$ called _____.

(3) Write Trapezoidal rule.

(4) Write Simpson's 1/3 rule.

(b) Answer any **one** : **2**

(1) Obtain central difference table for the following data :

| $x$ | 60 | 75 | 90 | 105 | 120 |
|---|---|---|---|---|---|
| $f(x)$ | 28.2 | 38.2 | 43.2 | 40.9 | 37.7 |

(2) Find the value of $\int_0^{10} \frac{dx}{1+x^2}$ by trapezoidal rule.

(c) Answer any **one** : **3**

(1) Explain Trapezoidal rule.

(2) Check : $D^3 = \frac{1}{h^3}\left[\Delta^3 - \frac{3}{2}\Delta^4 + \frac{7}{4}\Delta^5 + .....\right]$ or not.

(d) Answer any **one** : **5**

(1) Explain Simpson's 3/8 rule.

(2) Obtain derivatives using Stirling's formula.

**5** (a) Answer the following questions : **4**

(1) Write Range's formula for $K_0$.

(2) Write Range Kutta's formula to find $K_0$.

(3) Write Taylor's formula to solve ordinary differential equation.

(4) Write Picard's formula to solve ordinary differential equation.

(b) Answer any **one** : **2**

(1) Write two difference between Gauss-Backward interpolation and Lagrange's interpolation.

(2) Using Picard's method solve $\frac{dy}{dx} = x + y$, solve first approximation. Initial condition is $y(0) = 1$.

(c) Answer any **one** : **3**

(1) Explain Range's method.

(2) Explain Range Kutta method.

(d) Answer any **one** : **5**

(1) Solve $\frac{dy}{dx} = 2e^x - y, y(0.1) = 2.010, y(0.2) = 2.040, y(0.3) = 2.090$. Find $y(0.4)$ correct to three decimal places applying Milne's predictor method.

(2) Explain Euler's method to solve ordinary differential equation.

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